MIKE 21 Wave Modelling

MIKE 21 Spectral Waves FM

Short Description
MIKE 21 SW - SPECTRAL WAVE MODEL FM

MIKE 21 SW is a state-of-the-art third generation spectral wind-wave model developed by DHI. The model simulates the growth, decay and transformation of wind-generated waves and swells in offshore and coastal areas.

MIKE 21 SW includes two different formulations:
- Fully spectral formulation
- Directional decoupled parametric formulation

The fully spectral formulation is based on the wave action conservation equation, as described in e.g. Komen et al (1994) and Young (1999). The directional decoupled parametric formulation is based on a parameterisation of the wave action conservation equation. The parameterisation is made in the frequency domain by introducing the zeroth and first moment of the wave action spectrum. The basic conservation equations are formulated in either Cartesian co-ordinates for small-scale applications and polar spherical co-ordinates for large-scale applications.

The fully spectral model includes the following physical phenomena:
- Wave growth by action of wind
- Non-linear wave-wave interaction
- Dissipation due to white-capping
- Dissipation due to bottom friction
- Dissipation due to depth-induced wave breaking
- Refraction and shoaling due to depth variations
- Wave-current interaction
- Effect of time-varying water depth
- Effect of ice coverage on the wave field

The discretisation of the governing equation in geographical and spectral space is performed using cell-centred finite volume method. In the geographical domain, an unstructured mesh technique is used. The time integration is performed using a fractional step approach where a multi-sequence explicit method is applied for the propagation of wave action.

A MIKE 21 SW forecast application in the North Sea and Baltic Sea. The chart shows a wave field (from the NSBS model) illustrated by the significant wave height in top of the computational mesh. See also www.waterforecast.com
Computational Features
The main computational features of MIKE 21 SW - Spectral Wave Model FM are as follows:

- Fully spectral and directionally decoupled parametric formulations
- Source functions based on state-of-the-art 3rd generation formulations
- Instationary and quasi-stationary solutions
- Optimal degree of flexibility in describing bathymetry and ambient flow conditions using depth-adaptive and boundary-fitted unstructured mesh
- Coupling with hydrodynamic flow model for modelling of wave-current interaction and time-varying water depth
- Flooding and drying in connection with time-varying water depths
- Cell-centred finite volume technique
- Fractional step time-integration with an multi-sequence explicit method for the propagation
- Extensive range of model output parameters (wave, swell, air-sea interaction parameters, radiation stress tensor, spectra, etc.)

Application Areas
MIKE 21 SW is used for the assessment of wave climates in offshore and coastal areas - in hindcast and forecast mode.

A major application area is the design of offshore, coastal and port structures where accurate assessment of wave loads is of utmost importance to the safe and economic design of these structures.

In this case, the measured data can then be supplemented with hindcast data through the simulation of wave conditions during historical storms using MIKE 21 SW.

Example of a global application of MIKE 21 SW. The upper panel shows the bathymetry. Results from such a model (cf. lower panel) can be used as boundary conditions for regional scale forecast or hindcast models. See http://www.waterforecast.com for more details on regional and global modelling.

MIKE 21 SW is particularly applicable for simultaneous wave prediction and analysis on regional scale and local scale. Coarse spatial and temporal resolution is used for the regional part of the mesh and a high-resolution boundary and depth-adaptive mesh is describing the shallow water environment at the coastline.

Illustrations of typical application areas of DHI’s MIKE 21 SW – Spectral Wave Model FM

Measured data are often not available during periods long enough to allow for the establishment of sufficiently accurate estimates of extreme sea states.

Example of a computational mesh used for transformation of offshore wave statistics using the directionally decoupled parametric formulation
MIKE 21 SW is also used for the calculation of the sediment transport, which for a large part is determined by wave conditions and associated wave-induced currents. The wave-induced current is generated by the gradients in radiation stresses that occur in the surf zone.

MIKE 21 SW can be used to calculate the wave conditions and associated radiation stresses. The long-shore currents and sediment transport are then calculated using the flow and sediment transport models available in the MIKE 21 package. For such type of applications, the directional decoupled parametric formulation of MIKE 21 SW is an excellent compromise between the computational effort and accuracy.

Map of significant wave height (upper), current field (middle) and vector field (lower). The flow field is simulated by DHI’s MIKE 21 Flow Model FM, which is dynamically coupled to MIKE 21 SW
Model Equations

In MIKE 21 SW, the wind waves are represented by the wave action density spectrum \( N(\sigma, \theta) \). The independent phase parameters have been chosen as the relative (intrinsic) angular frequency, \( \sigma = 2\pi f \) and the direction of wave propagation, \( \theta \). The relation between the relative angular frequency and the absolute angular frequency, \( \omega \), is given by the linear dispersion relationship

\[
\sigma = \sqrt{gk \tanh(kd)} = \omega - \vec{k} \cdot \vec{U}
\]

where \( g \) is the acceleration of gravity, \( d \) is the water depth and \( \vec{U} \) is the current velocity vector and \( \vec{k} \) is the wave number vector with magnitude \( k \) and direction \( \theta \). The action density, \( N(\sigma, \theta) \), is related to the energy density \( E(\sigma, \theta) \) by

\[
N = \frac{E}{\sigma}
\]

Fully Spectral Formulation

The governing equation in MIKE 21 SW is the wave action balance equation formulated in either Cartesian or spherical co-ordinates. In horizontal Cartesian co-ordinates, the conservation equation for wave action reads

\[
\frac{\partial N}{\partial t} + \nabla \cdot (\vec{v}N) = \frac{S}{\sigma}
\]

where \( N(\vec{x}, \sigma, \theta, t) \) is the action density, \( t \) is the time, \( \vec{x} = (x, y) \) is the Cartesian co-ordinates, \( \vec{v} = (c_x, c_y, c_{\sigma}, c_{\theta}) \) is the propagation velocity of a wave group in the four-dimensional phase space \( \vec{x} \), \( \sigma \) and \( \theta \). \( S \) is the source term for energy balance equation. \( \nabla \) is the four-dimensional differential operator in the \( \vec{x} \), \( \sigma \), \( \theta \) space. The characteristic propagation speeds are given by the linear kinematic relationships

\[
(c_x, c_y) = \frac{d\vec{x}}{dt} = \vec{c}_g + \vec{U} = \frac{1}{2} \left(1 + \frac{2kd}{\sinh(2kd)}\right) \frac{\sigma}{k} + \vec{U}
\]

\[
c_{\sigma} = \frac{d\sigma}{dt} = \frac{\partial \sigma}{\partial \vec{d}} \left[ \frac{\partial \vec{d}}{\partial t} + \vec{U} \cdot \nabla \right] - c_g \vec{k} \cdot \frac{\partial \vec{U}}{\partial \vec{s}}
\]

\[
c_{\theta} = \frac{d\theta}{dt} = - \frac{1}{k} \left[ \frac{\partial \sigma}{\partial \vec{d}} \frac{\partial \vec{d}}{\partial \vec{m}} + \vec{k} \cdot \frac{\partial \vec{U}}{\partial \vec{m}} \right]
\]

Here, \( s \) is the space co-ordinate in wave direction \( \theta \) and \( m \) is a co-ordinate perpendicular to \( s \). \( \nabla_{\vec{x}} \) is the two-dimensional differential operator in the \( \vec{x} \)-space.

Source Functions

The source function term, \( S \), on the right hand side of the wave action conservation equation is given by

\[
S = S_{in} + S_{nl} + S_{ds} + S_{bot} + S_{surf}
\]

Here \( S_{in} \) represents the momentum transfer of wind energy to wave generation, \( S_{nl} \) the energy transfer due non-linear wave-wave interaction, \( S_{ds} \) the dissipation of wave energy due to white-capping (deep water wave breaking), \( S_{bot} \) the dissipation due to bottom friction and \( S_{surf} \) the dissipation of wave energy due to depth-induced breaking.

The default source functions \( S_{in}, S_{nl} \) and \( S_{ds} \) in MIKE 21 SW are similar to the source functions implemented in the WAM Cycle 4 model, see Komen et al (1994).

The wind input is based on Janssen's (1989, 1991) quasi-linear theory of wind-wave generation, where the momentum transfer from the wind to the sea not only depends on the wind stress, but also the sea state itself. The non-linear energy transfer (through the resonant four-wave interaction) is approximated by the DIA approach, Hasselmann et al (1985). The source function describing the dissipation due to white-capping is based on the theory of Hasselmann (1974) and Janssen (1989). The bottom friction dissipation is modelled using the approach by Johnson and Kofod-Hansen (2000), which depends on the wave and sediment properties. The source function describing the bottom-induced wave breaking is based on the well-proven approach of Battjes and Janssen (1978) and Eldeberky and Battjes (1996).

A detailed description of the various source functions is available in Komen et al (1994) and Sørensen et al (2003), which also includes the references listed above.
**Directional Decoupled Parametric Formulation**

The directionally decoupled parametric formulation is based on a parameterisation of the wave action conservation equation. Following Holthuijsen et al. (1989), the parameterisation is made in the frequency domain by introducing the zeroth and first moment of the wave action spectrum as dependent variables.

A similar formulation is used in the MIKE 21 NSW Near-shore Spectral Wind-Wave Model, which is one of the most popular models for wave transformation in coastal and shallow water environment. However, with MIKE 21 SW it is not necessary to set up a number of different orientated bathymetries to cover varying wind and wave directions.

The parameterisation leads to the following coupled equations

$$
\frac{\partial (m_0)}{\partial t} + \frac{\partial (c_n m_0)}{\partial x} + \frac{\partial (c_n m_0)}{\partial y} + \frac{\partial (c_n m_0)}{\partial \theta} = T_0
$$

$$
\frac{\partial (m_1)}{\partial t} + \frac{\partial (c_n m_1)}{\partial x} + \frac{\partial (c_n m_1)}{\partial y} + \frac{\partial (c_n m_1)}{\partial \theta} = T_1
$$

where \( m_0(x, y, \theta) \) and \( m_1(x, y, \theta) \) are the zeroth and first moment of the action spectrum \( N(x, y, \sigma, \theta) \), respectively. \( T_0(x, y, \theta) \) and \( T_1(x, y, \theta) \) are source functions based on the action spectrum. The moments \( m_n(x, y, \theta) \) are defined as

$$
m_n(x, y, \theta) = \int_{0}^{\infty} \omega^n N(x, y, \omega, \theta) d\omega
$$

The source functions \( T_0 \) and \( T_1 \) take into account the effect of local wind generation (stationary solution mode only) and energy dissipation due to bottom friction and wave breaking. The effects of wave-current interaction are also included. The source functions for the local wind generation are derived from empirical growth relations, see Johnson (1998) for details.

**Numerical Methods**

The frequency spectrum (fully spectral model only) is split into a prognostic part for frequencies lower than a cut-off frequency \( \sigma_{\text{max}} \) and an analytical diagnostic tail for the high-frequency part of the spectrum

$$
E(\sigma, \theta) = E(\sigma_{\text{max}}, \theta) \left( \frac{\sigma}{\sigma_{\text{max}}} \right)^{-m}
$$

where \( m \) is a constant (= 5) as proposed by Komen et al (1994).
Integrating the wave action conservation over an area \( A \), the frequency interval \( \Delta \sigma \) and the directional interval \( \Delta \theta_n \) gives

\[
\frac{\partial}{\partial t} \int_{A} \int_{\Delta \theta_n} \int_{\Delta \sigma} N d\Omega d\sigma d\theta - \int_{A} \int_{\Delta \theta_n} \int_{\Delta \sigma} \frac{S}{\sigma} d\Omega d\sigma d\theta = \int_{A} \int_{\Delta \theta_n} \int_{\Delta \sigma} \nabla \cdot (\mathbf{v} N) d\Omega d\sigma d\theta
\]

where \( \Omega \) is the integration variable defined on \( A \).

Using the divergence theorem and introducing the convective flux \( \mathbf{F} = \mathbf{v} N \), we obtain

\[
\frac{\partial N_{i,l,m}}{\partial t} = -\frac{1}{A} \sum_{p=1}^{NE} (F_n)_{p,l,m} \Delta l_p
\]

\[
-\frac{1}{\Delta \sigma_i} \left[ (F_\sigma)_{i,l+1/2,m} - (F_\sigma)_{i,l-1/2,m} \right] + \frac{S_{i,l,m}}{\sigma_i}
\]

where \( NE \) is the total number of edges in the cell, \((F_n)_{p,l,m} = (F_n^x + F_n^y)_{p,l,m}\) is the normal flux through the edge \( p \) in geographical space with length \( \Delta l_p \), \((F_\sigma)_{i,l+1/2,m} \) and \((F_\sigma)_{i,l-1/2,m} \) is the flux through the face in the frequency and directional space, respectively.

The convective flux is derived using a first-order upwinding scheme. In that

\[
F_n = c_n \left( \frac{1}{2} (N_i + N_j) - \frac{1}{2} \frac{c_n}{k} (N_i - N_j) \right)
\]

where \( c_n \) is the propagation speed normal to the element cell face.

**Time Integration**

The integration in time is based on a fractional step approach. Firstly, a propagation step is performed calculating an approximate solution \( N^* \) at the new time level \((n+1)\) by solving the homogenous wave action conservation equation, i.e. without the source terms. Secondly, a source terms step is performed calculating the new solution \( N^{n+1} \) from the estimated solution taking into account only the effect of the source terms.

The propagation step is carried out by an explicit Euler scheme

\[
N^*_{i,l,m} = N^n_{i,l,m} + \Delta t \left( \frac{\partial N_{i,l,m}}{\partial t} \right)^n
\]

To overcome the severe stability restriction, a multi-sequence integration scheme is employed. The maximum allowed time step is increased by employing a sequence of integration steps locally, where the number of steps may vary from point to point.

A source term step is performed using an implicit method (see Komen et al, 1994)

\[
N^{n+1}_{i,l,m} = N^*_{i,l,m} + \Delta t \left[ (1 - \alpha) S^*_ {i,l,m} + \alpha S^{n+1} {i,l,m} \right]
\]

where \( \alpha \) is a weighting coefficient that determines the type of finite difference method. Using a Taylor series to approximate \( S^{n+1} \) and assuming the off-diagonal terms in \( \partial N / \partial \sigma = \gamma \) are negligible, this equation can be simplified as

\[
N^{n+1}_{i,l,m} = N^n_{i,l,m} + \frac{S^*_ {i,l,m} l/\sigma_i) \Delta t}{1 - \alpha \gamma \Delta t}
\]

For growing waves (\( \gamma > 0 \)) an explicit forward difference is used (\( \alpha = 0 \)), while for decaying waves (\( \gamma < 0 \)) an implicit backward difference (\( \alpha = 1 \)) is applied.

MIKE 21 SW is also applied for wave forecasts in ship route planning and improved service for conventional and fast ferry operators.
Model Input
The necessary input data can be divided into following groups:

- Domain and time parameters:
  - computational mesh
  - co-ordinate type (Cartesian or spherical)
  - simulation length and overall time step

- Equations, discretisation and solution technique
  - formulation type
  - frequency and directional discretisation
  - number of time step groups
  - number of source time steps

- Forcing parameters
  - water level data
  - current data
  - wind data
  - ice data

- Source function parameters
  - non-linear energy transfer
  - wave breaking (shallow water)
  - bottom friction
  - white capping

- Structures
  - location and geometry
  - approach
  - structures coefficients

- Initial conditions
  - zero-spectrum (cold-start)
  - empirical data
  - data file

- Boundary conditions
  - closed boundaries
  - open boundaries (data format and type)

Providing MIKE 21 SW with a suitable mesh is essential for obtaining reliable results from the model. Setting up the mesh includes the appropriate selection of the area to be modelled, adequate resolution of the bathymetry, flow, wind and wave fields under consideration and definition of codes for essential and land boundaries.

Furthermore, the resolution in the geographical space must also be selected with respect to stability considerations.

As the wind is the main driving force in MIKE 21 SW, accurate hindcast or forecast wind fields are of utmost importance for the wave prediction.
Model Output
At each mesh point and for each time step four types of output can be obtained from MIKE 21 SW:

- Integral wave parameters divided into wind sea and swell such as
  - significant wave height, $H_{m0}$
  - peak wave period, $T_p$
  - averaged wave period, $T_{10}$
  - zero-crossing wave period, $T_{02}$
  - wave energy period, $T_01$
  - peak wave direction, $\theta_p$
  - mean wave direction, $\theta_m$
  - directional standard deviation, $\sigma$
  - wave height with dir., $H_m \cos \theta_m$, $H_m \sin \theta_m$
  - radiation stress tensor, $S_{xx}$, $S_{xy}$ and $S_{yy}$
  - particle velocities, horizontal/vertical
  - wave power, $P$, $P_x$, and $P_y$

Example of model output (directional-frequency wave spectrum) processed using the Polar Plot control in the MIKE Zero Plot Composer

![Polar Plot Example](image)

The distinction between wind-sea and swell can be calculated using either a constant threshold frequency or a dynamic threshold frequency with an upper frequency limit.

- **Input parameters**
  - water level, $WL$
  - water depth, $h$
  - current velocity, $\bar{U}$
  - wind speed, $U_{10}$
  - wind direction, $\theta_w$
  - ice concentration

- **Model parameters**
  - bottom friction coefficient, $C_f$
  - breaking parameter, $\gamma$
  - Courant number, $Cr$
  - time step factor, $\alpha$
  - characteristic edge length, $\Delta l$
  - area of element, $a$
  - wind friction speed, $u_*$
  - roughness length, $z_0$
  - drag coefficient, $C_D$
  - Charnock parameter, $z_{ch}$

- **Directional-frequency wave spectra at selected grid points and or areas as well as direction spectra and frequency spectra**

Output from MIKE 21 SW is typically post-processed using the Data Viewer available in the common MIKE Zero shell. The Data Viewer is a tool for analysis and visualisation of unstructured data, e.g. to view meshes, spectra, bathymetries, results files of different format with graphical extraction of time series and line series from plan view and import of graphical overlays.

Various other editors and plot controls in the MIKE Zero Composer (e.g. Time Series Plot, Polar Plot, etc.) can be used for analysis and visualisation.

![Data Viewer Example](image)

The Data Viewer in MIKE Zero – an efficient tool for analysis and visualisation of unstructured data including processing of animations.
Validation

The model has successfully been applied to a number of rather basic idealised situations for which the results can be compared with analytical solutions or information from the literature. The basic tests covered fundamental processes such as wave propagation, depth-induced and current-induced shoaling and refraction, wind-wave generation and dissipation.

Comparison between measured and simulated significant wave height, peak wave period and mean wave period at the Ekofisk offshore platform (water depth 70 m) in the North Sea. (▬▬) calculations and (▬▬) measurements

A major application area of MIKE 21 SW is in connection with design and maintenance of offshore structures

The model has also been tested in natural geophysical conditions (e.g. in the North Sea, the Danish West Coast and the Baltic Sea), which are more realistic and complicated than the academic test and laboratory tests mentioned above.

Comparison between measured and simulated significant wave height, peak wave period and mean wave period at Fjaltring located at the Danish west coast (water depth 17.5 m). (▬▬) calculations and (▬▬) measurements
The Fjaltring directional wave rider buoy is located offshore relative to the depicted arrow.

MIKE 21 SW is used for prediction of the wave conditions at the complex Horns Rev (reef) in the southeastern part of the North Sea. At this site, a 168 MW offshore wind farm with 80 turbines has been established in water depths between 6.5 and 13.5 m.

Comparison of frequency spectra at Fjaltring.
(▬▬) calculations and (───) measurements

The upper panels show the Horns Rev offshore wind farm and MIKE C-map chart. The middle panel shows a close-up of the mesh near the Horns Rev S wave rider buoy (10 m water depth. The lower panel shows a comparison between measured and simulated significant wave height at Horns Rev S, (▬▬) calculations including tide and surge and (───) calculations excluding including tide and surge, (o) measurements.

Comparison of frequency spectra at Fjaltring.
(▬▬) calculations and (───) measurements
The predicted nearshore wave climate along the island of Hiddensee and the coastline of Zingst located in the micro-tidal Gellen Bay, Germany have been compared to field measurements (Sørensen et al. 2004) provided by the MORWIN project. From the illustrations it can be seen that the wave conditions are well reproduced both offshore and in more shallow water near the shore. The RMS values (on significant wave height) are less than 0.25m at all five stations.

A MIKE 21 SW hindcast application in the Baltic Sea. The upper chart shows the bathymetry and the middle and lower charts show the computational mesh. The lower chart indicates the location of the measurement stations.

Time series of significant wave height, $H_m0$, peak wave period, $T_p$, and mean wave direction, MWD, at Darss sill (Offshore, depth 20.5 m). (▬▬) Calculation and (o) measurements. The RMS value on $H_m0$ is approximately 0.2 m.

Time series of significant wave height, $H_m0$, at Gellen (upper, depth 8.3m) and Bock (lower, depth 5.5 m). (▬▬) Calculation and (o) measurements. The RMS value on $H_m0$ is approximately 0.15 m.
The common MIKE Zero shell provides entries for common data file editors, plotting facilities and a toolbox for utilities as the Mesh Generator and Data Viewer.

**FEMA Approval of MIKE 21**

The US Federal Emergency Management Agency (FEMA) has on May 2001 officially approved MIKE 21 for use in coastal flood insurance studies.

The three modules, which are the hydro-dynamic module, non-linear spectral wind-wave module and offshore spectral wind-wave module, have been validated for coastal storm surge, coastal wave heights, and coastal wave effect groups.

For more information please contact: www.usa@vordanco and www.usa.vordanco.com.

**Overview of the common MIKE Zero utilities**
Parallelisation
The computational engines of the MIKE 21/3 FM series are available in versions that have been parallelised using both shared memory as well as distributed memory architecture. The latter approach allows for domain decomposition. The result is much faster simulations on systems with many cores.

Example of MIKE 21 HD FM speed-up using a HPC Cluster with distributed memory architecture (purple)

Hardware and Operating System Requirements
The MIKE Zero Modules support Microsoft Windows 7 Professional Service Pack 1 (64 bit), Windows 10 Pro (64 bit), Windows Server 2012 R2 Standard (64 bit) and Windows Server 2016 Standard (64 bit).

Microsoft Internet Explorer 9.0 (or higher) is required for network license management. An internet browser is also required for accessing the web-based documentation and online help.

The recommended minimum hardware requirements for executing the MIKE Zero modules are:

<table>
<thead>
<tr>
<th>Component</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processor</td>
<td>3 GHz PC (or higher)</td>
</tr>
<tr>
<td>Memory (RAM)</td>
<td>2 GB (or higher)</td>
</tr>
<tr>
<td>Hard disk</td>
<td>40 GB (or higher)</td>
</tr>
<tr>
<td>Monitor</td>
<td>SVGA, resolution 1024x768</td>
</tr>
<tr>
<td>Graphics card</td>
<td>64 MB RAM (256 MB RAM or</td>
</tr>
<tr>
<td>(GUI and visualisation)</td>
<td>higher is recommended)</td>
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</tbody>
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Support
News about new features, applications, papers, updates, patches, etc. are available here:


For further information on MIKE 21 SW, please contact your local DHI office or the support centre:

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Documentation
The MIKE 21 & MIKE 3 FM models are provided with comprehensive user guides, online help, scientific documentation, application examples and step-by-step training examples.
References


References on Applications

